

$$\begin{aligned}
 -F_D - w &= ma_y \\
 -0.0100v^2 - 98.0 &= 10.0a \\
 a &= -0.00100v^2 - 9.80.
 \end{aligned}$$

The acceleration depends on  $v$  and is therefore variable. Since  $a = f(v)$ , we can relate  $a$  to  $v$  using the rearrangement described above,

$$a \, ds = v \, dv.$$

We replace  $ds$  with  $dy$  because we are dealing with the vertical direction,

$$ady = vdv, \quad (-0.00100v^2 - 9.80)dy = vdv.$$

We now separate the variables ( $v$ 's and  $dv$ 's on one side;  $dy$  on the other):

$$\begin{aligned}
 \int_0^h dy &= \int_{50.0}^0 \frac{v \, dv}{(-0.00100v^2 - 9.80)} \\
 \int_0^h dy &= - \int_{50.0}^0 \frac{v \, dv}{(0.00100v^2 + 9.80)} = (-5 \times 10^3) \ln(0.00100v^2 + 9.80) \Big|_{50.0}^0.
 \end{aligned}$$

Thus,  $h = 114$  m.

### Significance

Notice the need to apply calculus since the force is not constant, which also means that acceleration is not constant. To make matters worse, the force depends on  $v$  (not  $t$ ), and so we must use the trick explained prior to the example. The answer for the height indicates a lower elevation if there were air resistance. We will deal with the effects of air resistance and other drag forces in greater detail in **Drag Force and Terminal Speed**.



**6.6 Check Your Understanding** If atmospheric resistance is neglected, find the maximum height for the mortar shell. Is calculus required for this solution?



Explore the forces at work in this **simulation** (<https://openstaxcollege.org//21forcesatwork>) when you try to push a filing cabinet. Create an applied force and see the resulting frictional force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

## 6.2 | Friction

### Learning Objectives

By the end of the section, you will be able to:

- Describe the general characteristics of friction
- List the various types of friction
- Calculate the magnitude of static and kinetic friction, and use these in problems involving Newton's laws of motion

When a body is in motion, it has resistance because the body interacts with its surroundings. This resistance is a force of friction. Friction opposes relative motion between systems in contact but also allows us to move, a concept that becomes obvious if you try to walk on ice. Friction is a common yet complex force, and its behavior still not completely understood.

Still, it is possible to understand the circumstances in which it behaves.

## Static and Kinetic Friction

The basic definition of **friction** is relatively simple to state.

### Friction

Friction is a force that opposes relative motion between systems in contact.

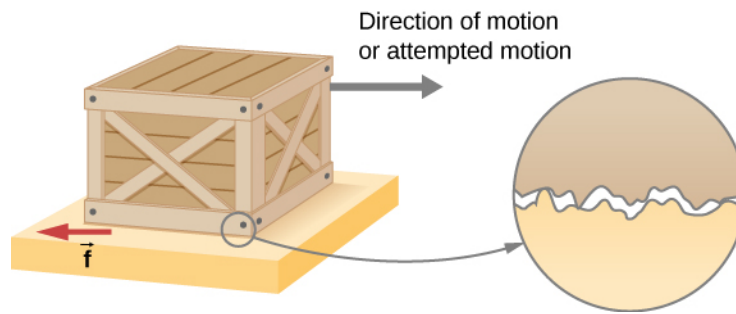
There are several forms of friction. One of the simpler characteristics of sliding friction is that it is parallel to the contact surfaces between systems and is always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. For example, friction slows a hockey puck sliding on ice. When objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between two objects.

### Static and Kinetic Friction

If two systems are in contact and stationary relative to one another, then the friction between them is called **static friction**. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you might push very hard on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. If you finally push hard enough, the crate seems to slip suddenly and starts to move. Now static friction gives way to kinetic friction. Once in motion, it is easier to keep it in motion than it was to get it started, indicating that the kinetic frictional force is less than the static frictional force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it easier to get the crate started and keep it going (as you might expect).

**Figure 6.10** is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. Thus, when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, breaking off the points, or both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explains the dependence of friction on the nature of the substances. For example, rubber-soled shoes slip less than those with leather soles. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



**Figure 6.10** Frictional forces, such as  $\vec{f}$ , always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. For the object to move, it must rise to where the peaks of the top surface can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. (In fact, perfectly smooth, clean surfaces of similar materials would adhere, forming a bond called a “cold weld.”)

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for situations involving motion (kinetic friction). What follows is an approximate empirical (experimentally determined) model only. These equations for static and kinetic friction are not vector equations.

### Magnitude of Static Friction

The magnitude of static friction  $f_s$  is

$$f_s \leq \mu_s N, \quad (6.1)$$

where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force.

The symbol  $\leq$  means *less than or equal to*, implying that static friction can have a maximum value of  $\mu_s N$ . Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds

$f_s(\text{max})$ , the object moves. Thus,

$$f_s(\text{max}) = \mu_s N.$$

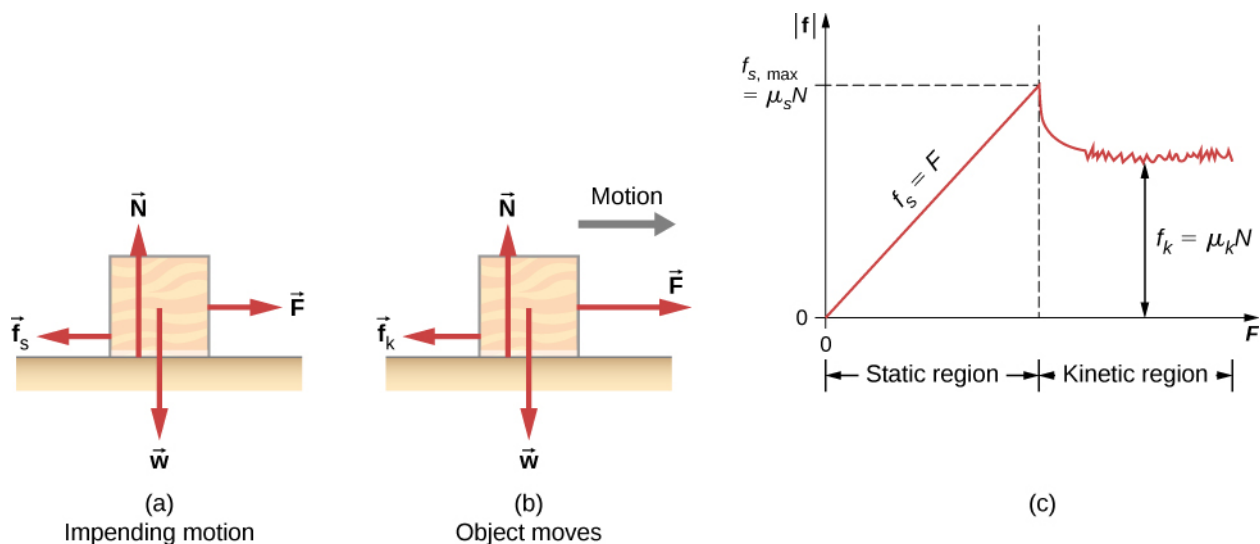
### Magnitude of Kinetic Friction

The magnitude of kinetic friction  $f_k$  is given by

$$f_k = \mu_k N, \quad (6.2)$$

where  $\mu_k$  is the coefficient of kinetic friction.

A system in which  $f_k = \mu_k N$  is described as a system in which *friction behaves simply*. The transition from static friction to kinetic friction is illustrated in **Figure 6.11**.



**Figure 6.11** (a) The force of friction  $\vec{f}$  between the block and the rough surface opposes the direction of the applied force  $\vec{F}$ . The magnitude of the static friction balances that of the applied force. This is shown in the left side of the graph in (c). (b) At some point, the magnitude of the applied force is greater than the force of kinetic friction, and the block moves to the right. This is shown in the right side of the graph. (c) The graph of the frictional force versus the applied force; note that  $f_s(\text{max}) > f_k$ . This means that  $\mu_s > \mu_k$ .

As you can see in **Table 6.1**, the coefficients of kinetic friction are less than their static counterparts. The approximate values of  $\mu$  are stated to only one or two digits to indicate the approximate description of friction given by the preceding two equations.

System	Static Friction $\mu_s$	Kinetic Friction $\mu_k$
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.5-0.7	0.3-0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

**Table 6.1** Approximate Coefficients of Static and Kinetic Friction

**Equation 6.1** and **Equation 6.2** include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force is equal to its weight,

$$w = mg = (100 \text{ kg})(9.80 \text{ m/s}^2) = 980 \text{ N},$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

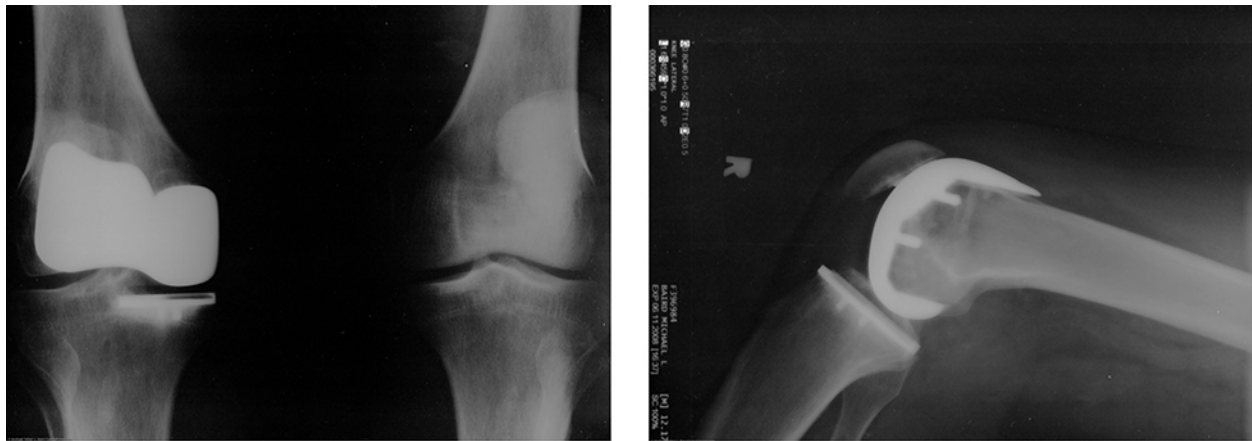
$$f_s(\text{max}) = \mu_s N = (0.45)(980 \text{ N}) = 440 \text{ N}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only

$$f_k = \mu_k N = (0.30)(980 \text{ N}) = 290 \text{ N}$$

keeps it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unitless quantity with a magnitude usually between 0 and 1.0. The actual value depends on the two surfaces that are in contact.

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost-glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (**Figure 6.12**). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



**Figure 6.12** Artificial knee replacement is a procedure that has been performed for more than 20 years. These post-operative X-rays show a right knee joint replacement. (credit: modification of work by Mike Baird)

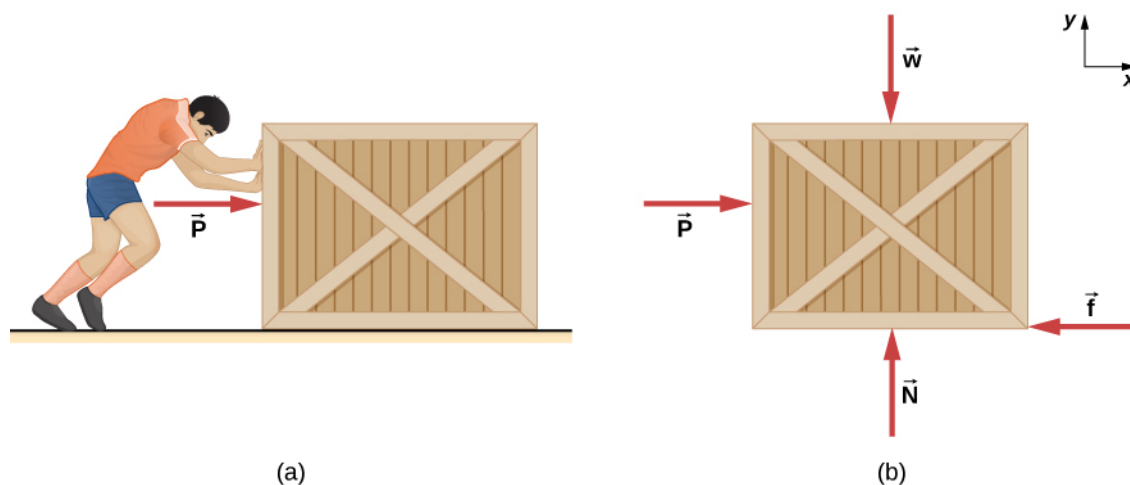
Natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Hospitals and doctor's clinics commonly use artificial lubricants, such as gels, to reduce friction.

The equations given for static and kinetic friction are empirical laws that describe the behavior of the forces of friction. While these formulas are very useful for practical purposes, they do not have the status of mathematical statements that represent general principles (e.g., Newton's second law). In fact, there are cases for which these equations are not even good approximations. For instance, neither formula is accurate for lubricated surfaces or for two surfaces sliding across each other at high speeds. Unless specified, we will not be concerned with these exceptions.

### Example 6.10

#### Static and Kinetic Friction

A 20.0-kg crate is at rest on a floor as shown in **Figure 6.13**. The coefficient of static friction between the crate and floor is 0.700 and the coefficient of kinetic friction is 0.600. A horizontal force  $\vec{P}$  is applied to the crate. Find the force of friction if (a)  $\vec{P} = 20.0 \text{ N}$ , (b)  $\vec{P} = 30.0 \text{ N}$ , (c)  $\vec{P} = 120.0 \text{ N}$ , and (d)  $\vec{P} = 180.0 \text{ N}$ .



**Figure 6.13** (a) A crate on a horizontal surface is pushed with a force  $\vec{P}$ . (b) The forces on the crate. Here,  $\vec{f}$  may represent either the static or the kinetic frictional force.

### Strategy

The free-body diagram of the crate is shown in **Figure 6.13**(b). We apply Newton's second law in the horizontal and vertical directions, including the friction force in opposition to the direction of motion of the box.

### Solution

Newton's second law GIVES

$$\begin{aligned}\sum F_x &= ma_x & \sum F_y &= ma_y \\ P - f &= ma_x & N - w &= 0.\end{aligned}$$

Here we are using the symbol  $f$  to represent the frictional force since we have not yet determined whether the crate is subject to static friction or kinetic friction. We do this whenever we are unsure what type of friction is acting. Now the weight of the crate is

$$w = (20.0 \text{ kg})(9.80 \text{ m/s}^2) = 196 \text{ N},$$

which is also equal to  $N$ . The maximum force of static friction is therefore  $(0.700)(196 \text{ N}) = 137 \text{ N}$ . As long as  $\vec{P}$  is less than 137 N, the force of static friction keeps the crate stationary and  $f_s = \vec{P}$ . Thus, (a)  $f_s = 20.0 \text{ N}$ , (b)  $f_s = 30.0 \text{ N}$ , and (c)  $f_s = 120.0 \text{ N}$ .

(d) If  $\vec{P} = 180.0 \text{ N}$ , the applied force is greater than the maximum force of static friction (137 N), so the crate can no longer remain at rest. Once the crate is in motion, kinetic friction acts. Then

$$f_k = \mu_k N = (0.600)(196 \text{ N}) = 118 \text{ N},$$

and the acceleration is

$$a_x = \frac{\vec{P} - f_k}{m} = \frac{180.0 \text{ N} - 118 \text{ N}}{20.0 \text{ kg}} = 3.10 \text{ m/s}^2.$$

### Significance

This example illustrates how we consider friction in a dynamics problem. Notice that static friction has a value that matches the applied force, until we reach the maximum value of static friction. Also, no motion can occur until the applied force equals the force of static friction, but the force of kinetic friction will then become smaller.



**6.7 Check Your Understanding** A block of mass 1.0 kg rests on a horizontal surface. The frictional coefficients for the block and surface are  $\mu_s = 0.50$  and  $\mu_k = 0.40$ . (a) What is the minimum horizontal force required to move the block? (b) What is the block's acceleration when this force is applied?

## Friction and the Inclined Plane

One situation where friction plays an obvious role is that of an object on a slope. It might be a crate being pushed up a ramp to a loading dock or a skateboarder coasting down a mountain, but the basic physics is the same. We usually generalize the sloping surface and call it an inclined plane but then pretend that the surface is flat. Let's look at an example of analyzing motion on an inclined plane with friction.

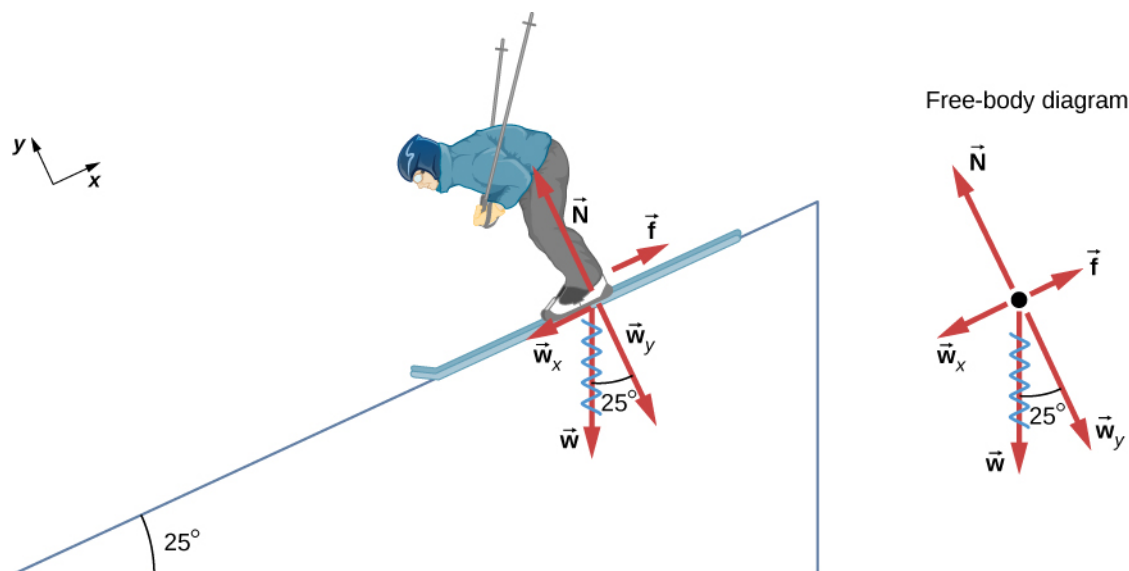
### Example 6.11

#### Downhill Skier

A skier with a mass of 62 kg is sliding down a snowy slope at a constant velocity. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

#### Strategy

The magnitude of kinetic friction is given as 45.0 N. Kinetic friction is related to the normal force  $N$  by  $f_k = \mu_k N$ ; thus, we can find the coefficient of kinetic friction if we can find the normal force on the skier. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See **Figure 6.14**, which repeats a figure from the chapter on Newton's laws of motion.)



**Figure 6.14** The motion of the skier and friction are parallel to the slope, so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). The normal force  $\vec{N}$  is perpendicular to the slope, and friction  $\vec{f}$  is parallel to the slope, but the skier's weight  $\vec{w}$  has components along both axes, namely  $\vec{w}_y$  and  $\vec{w}_x$ . The normal force  $\vec{N}$  is equal in magnitude to  $\vec{w}_y$ , so there is no motion perpendicular to the slope. However,  $\vec{f}$  is equal to  $\vec{w}_x$  in magnitude, so there is a constant velocity down the slope (along the  $x$ -axis).

We have

$$N = w_y = w \cos 25^\circ = mg \cos 25^\circ.$$

Substituting this into our expression for kinetic friction, we obtain

$$f_k = \mu_k mg \cos 25^\circ,$$

which can now be solved for the coefficient of kinetic friction  $\mu_k$ .

### Solution

Solving for  $\mu_k$  gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w \cos 25^\circ} = \frac{f_k}{mg \cos 25^\circ}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0 \text{ N}}{(62 \text{ kg})(9.80 \text{ m/s}^2)(0.906)} = 0.082.$$

### Significance

This result is a little smaller than the coefficient listed in **Table 6.1** for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass  $m$  slides down a slope that makes an angle  $\theta$  with the horizontal, friction is given by  $f_k = \mu_k mg \cos \theta$ .

All objects slide down a slope with constant acceleration under these circumstances.

We have discussed that when an object rests on a horizontal surface, the normal force supporting it is equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force. When an object is not on a horizontal surface, as with the inclined plane, we must find the force acting on the object that is directed perpendicular to the surface; it is a component of the weight.

We now derive a useful relationship for calculating coefficient of friction on an inclined plane. Notice that the result applies only for situations in which the object slides at constant speed down the ramp.

An object slides down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in **Example 6.10**, the kinetic friction on a slope is  $f_k = \mu_k mg \cos \theta$ . The component of the weight down the slope is equal to  $mg \sin \theta$  (see the free-body diagram in **Figure 6.14**). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out,

$$\mu_k mg \cos \theta = mg \sin \theta.$$

Solving for  $\mu_k$ , we find that

$$\mu_k = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta.$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find  $\mu_k$ . Note that the coin does not start to slide at all until an angle greater than  $\theta$  is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Think about how this may affect the value for  $\mu_k$  and its uncertainty.

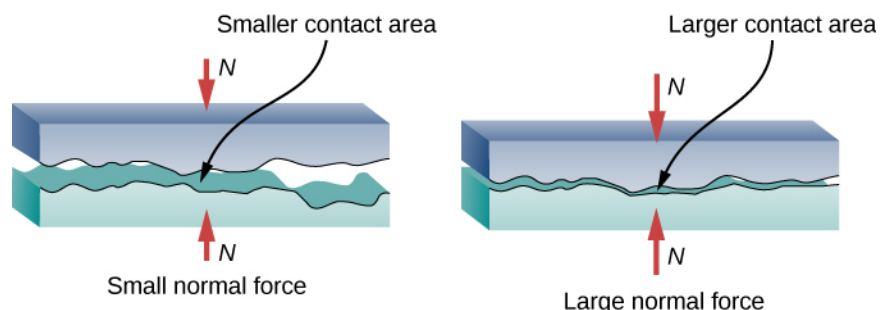
## Atomic-Scale Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) into heat.

**Figure 6.15** illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the amount of area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area

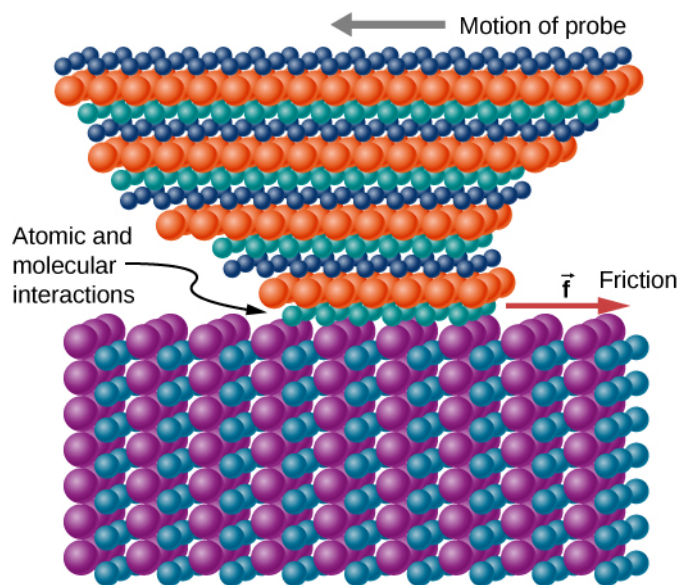


because only high spots touch. When a greater normal force is exerted, the actual contact area increases, and we find that the friction is proportional to this area.




**Figure 6.15** Two rough surfaces in contact have a much smaller area of actual contact than their total area. When the normal force is larger as a result of a larger applied force, the area of actual contact increases, as does friction.

However, the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance, and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. **Figure 6.16** shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which is discussed in **Static Equilibrium and Elasticity**. The variation in shear stress is remarkable (more than a factor of  $10^{12}$ ) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



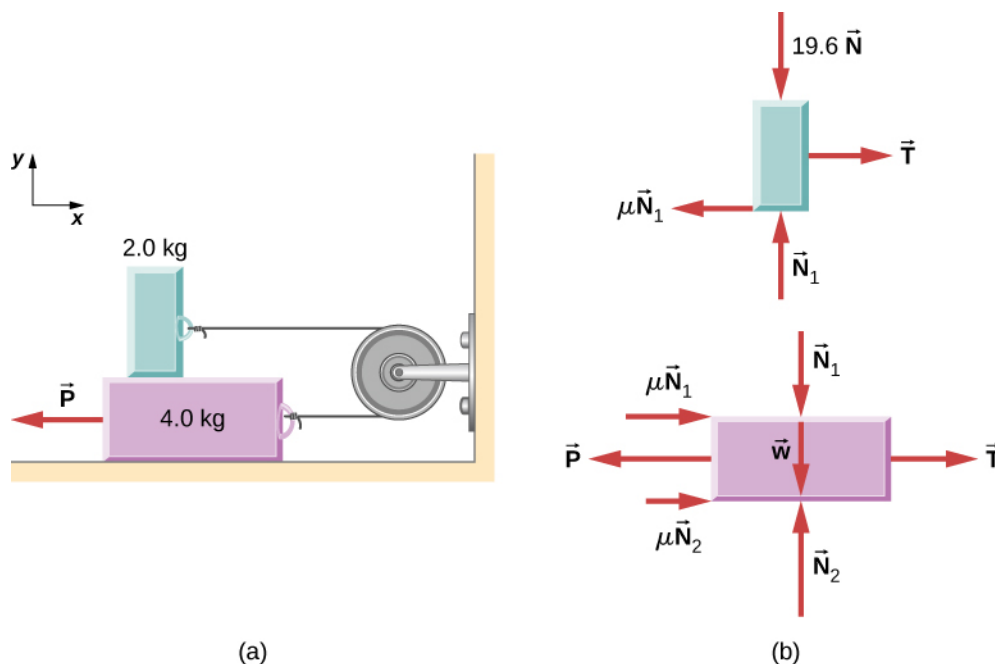
**Figure 6.16** The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

 Describe a **model for friction** (<https://openstaxcollege.org//21friction>) on a molecular level. Describe matter in terms of molecular motion. The description should include diagrams to support the description; how the temperature affects the image; what are the differences and similarities between solid, liquid, and gas particle motion; and how the size and speed of gas molecules relate to everyday objects.

## Example 6.12

### Sliding Blocks

The two blocks of **Figure 6.17** are attached to each other by a massless string that is wrapped around a frictionless pulley. When the bottom 4.00-kg block is pulled to the left by the constant force  $\vec{P}$ , the top 2.00-kg block slides across it to the right. Find the magnitude of the force necessary to move the blocks at constant speed. Assume that the coefficient of kinetic friction between all surfaces is 0.400.



**Figure 6.17** (a) Each block moves at constant velocity. (b) Free-body diagrams for the blocks.

### Strategy

We analyze the motions of the two blocks separately. The top block is subjected to a contact force exerted by the bottom block. The components of this force are the normal force  $N_1$  and the frictional force  $-0.400N_1$ . Other forces on the top block are the tension  $T_i$  in the string and the weight of the top block itself, 19.6 N. The bottom block is subjected to contact forces due to the top block and due to the floor. The first contact force has components  $-N_1$  and  $0.400N_1$ , which are simply reaction forces to the contact forces that the bottom block exerts on the top block. The components of the contact force of the floor are  $N_2$  and  $0.400N_2$ . Other forces on this block are  $-P$ , the tension  $T_i$ , and the weight  $-39.2$  N.

### Solution

Since the top block is moving horizontally to the right at constant velocity, its acceleration is zero in both the horizontal and the vertical directions. From Newton's second law,

$$\begin{aligned} \sum F_x &= m_1 a_x & \sum F_y &= m_1 a_y \\ T - 0.400N_1 &= 0 & N_1 - 19.6 \text{ N} &= 0. \end{aligned}$$

Solving for the two unknowns, we obtain  $N_1 = 19.6$  N and  $T = 0.40N_1 = 7.84$  N. The bottom block is also not accelerating, so the application of Newton's second law to this block gives

$$\begin{aligned} \sum F_x &= m_2 a_x & \sum F_y &= m_2 a_y \\ T - P + 0.400 N_1 + 0.400 N_2 &= 0 & N_2 - 39.2 \text{ N} - N_1 &= 0. \end{aligned}$$

The values of  $N_1$  and  $T$  were found with the first set of equations. When these values are substituted into the second set of equations, we can determine  $N_2$  and  $P$ . They are

$$N_2 = 58.8 \text{ N and } P = 39.2 \text{ N.}$$

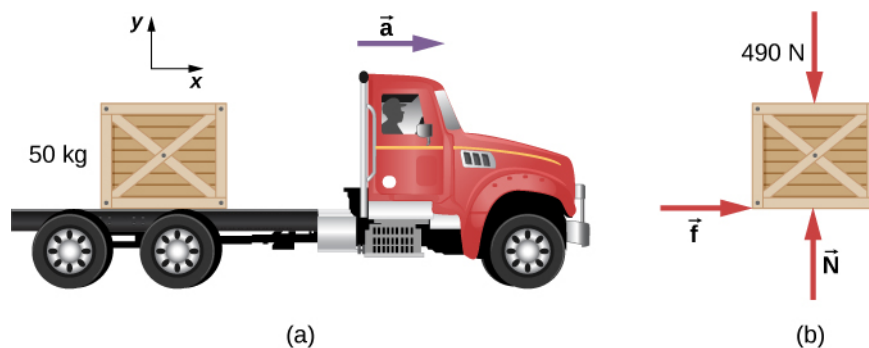
### Significance

Understanding what direction in which to draw the friction force is often troublesome. Notice that each friction force labeled in **Figure 6.17** acts in the direction opposite the motion of its corresponding block.

## Example 6.13

### A Crate on an Accelerating Truck

A 50.0-kg crate rests on the bed of a truck as shown in **Figure 6.18**. The coefficients of friction between the surfaces are  $\mu_k = 0.300$  and  $\mu_s = 0.400$ . Find the frictional force on the crate when the truck is accelerating forward relative to the ground at (a)  $2.00 \text{ m/s}^2$ , and (b)  $5.00 \text{ m/s}^2$ .



**Figure 6.18** (a) A crate rests on the bed of the truck that is accelerating forward. (b) The free-body diagram of the crate.

### Strategy

The forces on the crate are its weight and the normal and frictional forces due to contact with the truck bed. We start by *assuming* that the crate is not slipping. In this case, the static frictional force  $f_s$  acts on the crate. Furthermore, the accelerations of the crate and the truck are equal.

### Solution

- a. Application of Newton's second law to the crate, using the reference frame attached to the ground, yields

$$\begin{aligned} \sum F_x &= ma_x & \sum F_y &= ma_y \\ f_s &= (50.0 \text{ kg})(2.00 \text{ m/s}^2) & N - 4.90 \times 10^2 \text{ N} &= (50.0 \text{ kg})(0) \\ &= 1.00 \times 10^2 \text{ N} & N &= 4.90 \times 10^2 \text{ N.} \end{aligned}$$

We can now check the validity of our no-slip assumption. The maximum value of the force of static friction is

$$\mu_s N = (0.400)(4.90 \times 10^2 \text{ N}) = 196 \text{ N,}$$

whereas the *actual* force of static friction that acts when the truck accelerates forward at  $2.00 \text{ m/s}^2$  is only  $1.00 \times 10^2 \text{ N}$ . Thus, the assumption of no slipping is valid.

- b. If the crate is to move with the truck when it accelerates at  $5.00 \text{ m/s}^2$ , the force of static friction must be

$$f_s = ma_x = (50.0 \text{ kg})(5.00 \text{ m/s}^2) = 250 \text{ N}.$$

Since this exceeds the maximum of 196 N, the crate must slip. The frictional force is therefore kinetic and is

$$f_k = \mu_k N = (0.300)(4.90 \times 10^2 \text{ N}) = 147 \text{ N}.$$

The horizontal acceleration of the crate relative to the ground is now found from

$$\begin{aligned} \sum F_x &= ma_x \\ 147 \text{ N} &= (50.0 \text{ kg})a_x, \\ \text{so } a_x &= 2.94 \text{ m/s}^2. \end{aligned}$$

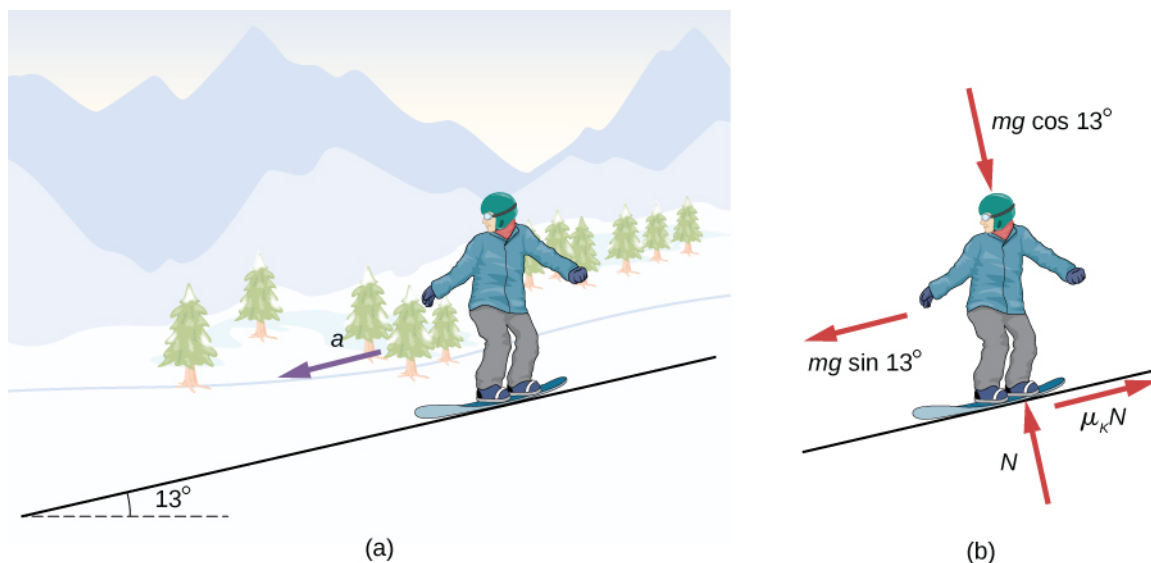
### Significance

Relative to the ground, the truck is accelerating forward at  $5.0 \text{ m/s}^2$  and the crate is accelerating forward at  $2.94 \text{ m/s}^2$ . Hence the crate is sliding backward relative to the bed of the truck with an acceleration  $2.94 \text{ m/s}^2 - 5.00 \text{ m/s}^2 = -2.06 \text{ m/s}^2$ .

## Example 6.14

### Snowboarding

Earlier, we analyzed the situation of a downhill skier moving at constant velocity to determine the coefficient of kinetic friction. Now let's do a similar analysis to determine acceleration. The snowboarder of **Figure 6.19** glides down a slope that is inclined at  $\theta = 13^\circ$  to the horizontal. The coefficient of kinetic friction between the board and the snow is  $\mu_k = 0.20$ . What is the acceleration of the snowboarder?



**Figure 6.19** (a) A snowboarder glides down a slope inclined at  $13^\circ$  to the horizontal. (b) The free-body diagram of the snowboarder.

### Strategy

The forces acting on the snowboarder are her weight and the contact force of the slope, which has a component normal to the incline and a component along the incline (force of kinetic friction). Because she moves along the

slope, the most convenient reference frame for analyzing her motion is one with the  $x$ -axis along and the  $y$ -axis perpendicular to the incline. In this frame, both the normal and the frictional forces lie along coordinate axes, the components of the weight are  $mg \sin \theta$  along the slope and  $mg \cos \theta$  at right angles into the slope, and the only acceleration is along the  $x$ -axis ( $a_y = 0$ ).

### Solution

We can now apply Newton's second law to the snowboarder:

$$\begin{aligned}\sum F_x &= ma_x & \sum F_y &= ma_y \\ mg \sin \theta - \mu_k N &= ma_x & N - mg \cos \theta &= m(0).\end{aligned}$$

From the second equation,  $N = mg \cos \theta$ . Upon substituting this into the first equation, we find

$$\begin{aligned}a_x &= g(\sin \theta - \mu_k \cos \theta) \\ &= g(\sin 13^\circ - 0.20 \cos 13^\circ) = 0.29 \text{ m/s}^2.\end{aligned}$$

### Significance

Notice from this equation that if  $\theta$  is small enough or  $\mu_k$  is large enough,  $a_x$  is negative, that is, the snowboarder slows down.



**6.8 Check Your Understanding** The snowboarder is now moving down a hill with incline  $10.0^\circ$ . What is the skier's acceleration?

## 6.3 | Centripetal Force

### Learning Objectives

By the end of the section, you will be able to:

- Explain the equation for centripetal acceleration
- Apply Newton's second law to develop the equation for centripetal force
- Use circular motion concepts in solving problems involving Newton's laws of motion

In **Motion in Two and Three Dimensions**, we examined the basic concepts of circular motion. An object undergoing circular motion, like one of the race cars shown at the beginning of this chapter, must be accelerating because it is changing the direction of its velocity. We proved that this centrally directed acceleration, called centripetal acceleration, is given by the formula

$$a_c = \frac{v^2}{r}$$

where  $v$  is the velocity of the object, directed along a tangent line to the curve at any instant. If we know the angular velocity  $\omega$ , then we can use

$$a_c = r\omega^2.$$

Angular velocity gives the rate at which the object is turning through the curve, in units of rad/s. This acceleration acts along the radius of the curved path and is thus also referred to as a radial acceleration.

An acceleration must be produced by a force. Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge. Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law